Dynamics of Large-Scale Atmospheric Flows

# Basics

## Notation

## Coordinate system

### Spherical coordinate system

### Unit vectors on the sphere

### Wind vectors on the sphere

## Basic equations

6 equations for 6 unknowns

### **Equations of motion (Navier-Stokes)**

are spherical unit coordinates (eastward, northward, vertical). Equations are simplified through -plane.

#### -plane approximation

,   
a = Earth radius

### **Continuity equation**

### **Equation of state**

### **Thermodynamic equation**

## Large-scale approximations (Synoptic-Scale Motions)

Large-scale approximations hold for the sea breeze and cumulus scale.

### Vertical component decomposition

Due to the strong vertical stratification of the atmosphere, it is useful to decompose a field variable as follows.

At a certain height,

This holds for

### Equation of state approx

### Potential temperature approx

### Vertical momentum equation approx

### **Hydrostatic approximation**

Using: Vertical momentum equation

### **Geostrophic wind**

**Using: Horizontal momentum equation**

**Equivalently,**

* **Geostrophic wind field is non-divergent**
* **Geostrophic approximation not valid for large Rossby numbers (Ro >> 1)**

### **Thermal wind**

**Using: Hydrostatic & geostrophic approximations & equation of state**

* **Left turning of geostrophic wind => Cold air advection**
* **Right turning of geostrophic wind => Warm air advection**

**Aka thermal wind relationship is the following**

# Vorticity

Vertical component of vorticity

## Vorticity equation

## Synoptic scale approximation

Meridional excursion

Flow convergence / divergence

### Assumptions

* Invalid in frontal regions with
* Caution: in tropical and equatorial regions
* If , then the vorticity equation is reduced to the **barotropic vorticity equation**.

# Quasi-geostrophic (QG) approximation

## Simplifications

Subscript G denotes the usage of the geostrophic wind, i.e.

## QG Equations

|  |  |
| --- | --- |
| Geostrophic approximation |  |
| Hydrostatic approximation |  |
| Vorticity equation |  |
| Mass conservation |  |
| Thermodynamic equation |  |

## Implications on vorticity / westward tilt

* Positive / negative values of are associated with low / high pressure
* Positive / negative values of are associated with low / high temperature
* Both effects together lead to the westward slope of cyclones and anticyclones.

## Ageostrophic wind

Split velocity in basic state and perturbation. The geostrophic wind is the basic state while the ageostrophic wind is the perturbation.

### Solution of QG inconsistency

Geostrophic approximation , but Mass conservation

Solution: where , so

### Some properties

## Linkage of ageo- and geostrophic wind

Assumptions: , , =const.

From thermodynamic equation and vorticity equation using hydrostatic approximation.

## Diagnostic equation (vertical wind)

With

**Ageostrophic wind is completely determined by geostrophic wind!**

## 4-step golden rule

1. W-equation
2. Combination of vorticity equation and mass conservation
3. Approximating with ?

### Finding

1. Find largest Q-vector. Arrowhead is zone of convergence (, tail is zone of divergence
2. Find largest temperature gradient and a strong wind change along the isentrope

## QG Potential Vorticity

1. Combination of vorticity equation and mass conservation.
2. Combine this with thermodynamic equation
3. Interchange and

* q = relative vorticity + static stability
  + Static stability: From hydrostatic equation:
* Only valid for adiabatic flow.
* Boundary condition:

### QG Prognostic system

1. known with B.C.
2. Main diagnostic step
   1. Solve
   2. yields and
   3. These yield the Q-vectors
   4. The Q-vectors yield
3. Do prognostic step using passive advection of and go to 2.

### QGPV key properties

* QG flow determined by interior distribution of QGPV and surface distribution of
* Conservation:
* Invertibility:
* Partition and attribution:

### Prototype vortex for QG flow

Since QGPV can be decomposed into “atomic” vortices, we can analyse prototype vortices.

Given:

Then:

* Here, r is the spherical symmetrical coordinate, and R is the “x” coordinate

## Ertel PV

Idea: Analogue derivation to QGPV, but starting from full NSE.

Applicable only for inviscid and adiabatic flow.

* absolute vorticity vector

## Isentropic PV

* IPV = z-component of PV

## Preliminaries for PV chart analysis

* Unit for PV: 1 pvu. 2 pvu are defined as the tropopause.
* increases towards equator.
* Max(PV) at equator due to Coriolis parameter.
* , because
* For adiabatic and inviscid flow

### PV anomaly / cyclogenesis

* Adiabatic movement of air parcel towards south => stays on isentrope.
* As it must retain PV, it “drags” its PV value along the isentrope, especially when intersecting the 2 pvu tropopause. A PV anomaly occurs.
* Because of far-field effect of a strong PV gradient (=> strong wind), a cyclone forms at ground.



## Idealised PV situations

Effect: Strong PV gradient w.r.t latitude => Strong wind.  
Reason: High PV => CCW flow, low PV => CW flow. Strongest wind at strongest gradient. C.f. Jetstream and PV

Effect: Positive / negative upper-level PV anomaly CCW / CW circulation and cold / warm below, warm / cold above  
Reason: Positive upper-level PV anomaly => higher (=> CCW flow ) and lower

Effect: Positive / negative surface temp. anomaly CCW / CW circulation and warm / cold above  
Reason: Negative sfc temp: Isentropes squeezed, higher => lower => CW flow

Effect: Diabatically produce low-level PV anomalies  
Reason: Microphysical processes

Effect: PV tower. If all anomalies are aligned on top of each other, they intensify and form a devastating storm.

## Diabatic PV

**Important**: is only proportional to , not

Insert: Image with diabatic PV through a cloud

# QG Wave theory

## Examples

* Large scale flows in mid and upper-troposphere
* Planetary scale quasi-stationary and quasi-steady wave features in stratosphere

## Waves on a uniform zonal flow

* Set:
* Split quantities in basic state and perturbations

From the QG potential vorticity equation, inserting and linearizing (i.e. discarding terms like ) yields the perturbation equation:

* , and , thus
* QGPV definition

### 2d wave solution (aka Rossby wave)

Solution on a mid-latitude band (all longitudes, latitudes 30-90°N)

### Dispersion relationship

* + East-West wavelength:
  + m=1 => One full sine period
  + North-South half-wavelength:
  + m’=1 => Half sine period

### Phase velocities

#### Effects

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
|  | -35.5 | -18.5 | -6.0 | -1.7 |

* **Short waves (large m) typically move with**
* **Long waves (small m) move against**

### Group velocities

#### Effects

* Longitudinal waves propagate eastwards
* Latitudinal waves propagate westwards

### More effects

Westward translation of a sinusoidal displacement of a fluid

### 3d wave solution

Yields a slightly different perturbation equation with solutions of the form:

With the following constraints

#### Effect: Vertical propagability

* for
* for

Discard exponential solution. Vertically propagating waves only occur for

* (: Wave velocity must be easterly relative to zonal flow
* (: For typical values of
* The above criteria are only satisfied for low zonal wavenumbers , since

|  |  |
| --- | --- |
| Zonal wavenumber |  |
| m=1 |  |
| m=2 |  |
| m=3 |  |

Real effect: **From the troposphere, only m=1 or m=2 waves propagate to the stratosphere.**

### Orographic forcing

Setting: Stationary, incompressible wave (, subject to sinusoidal terrain

**PV equation**

**Terrain () and Thermodynamic B.C.**

"

**Solution**

**There are large Rossby waves induced by (large) mountains such as the Rocky mountains, Himalaya or Greenland.**

**They propagate vertically for low wavenumbers (m <= 3) and decay vertically for high wavenumbers (m > 3)**

### Diabatic forcing

Large planetary scale diabatic heating distributions generate **vertically propagating waves at high altitudes. At the surface, the low and high pressure centres are displaced about ¼ wavelength eastward from centres of the diabatically heated and cooled regions.**

# Baroclinic instability



Baroclinic atmosphere

## Eady problem

Set: , where = height of atmosphere

**PV equation**

**Thermodynamic boundary condition**

## Ansatz

## Solution

Where

Where

Where

### Exponential growth solution

If , then

for



Eady growth rate

## Nature of solution

* Wavelength ( for
* Wavelength for
* Growth rate for
* **All perturbations of wavelength greater than 2500 km are unstable**

# Notions

* Cyclonic = Counter clockwise = Left turn
* Anticyclonic = Clockwise = Right turn
* Zonal wavenumber = m
* North-south petal count = m’
* Large waves ⬄ small m
* Short waves ⬄ large m

# Numerical values

|  |  |  |
| --- | --- | --- |
| Earth radius |  |  |
| Gas constant of air |  |  |
| Specific heat at constant volume |  |  |
| Specific heat at constant pressure |  |  |
| Coriolis parameter on mid-latitude |  |  |

# Quantities

## Coriolis parameter

## Full form

Rotation rate of Earth  
 Latitude

### -plane approximation

a = radius of earth

for mid-latitudes

for the Northern hemisphere

for the Southern hemisphere

## Brunt-Vaisala frequency

## Rossby number

Ratio of horizontal advection to Coriolis term

## Geopotential height

**Geopotential**

:latitude ,: geometric height

The geopotential could also be expressed as a function of pressure

**Geopotential height**

* As a function of geometric height  
  *:* Standard gravity at mean sea level
* As a function of pressure
* The geostrophic wind is parallel to the contours, and its magnitude is proportional to the distance between the contours.

## Potential temperature

Temperature which an air parcel would acquire if adiabatically brought from level 1 to level 0.

,

* Stable atmosphere
* Unstable atmosphere

## Circulation and vorticity

### Vorticity

### Mean vorticity

### Circulation

# Mathematical tricks

## Interchange of and

## Product rule reversed

The product rule is normally used to expand a derivative of a product to a product of derivatives. This can be done the other way around, too.

## Neglecting small terms

where and is an arbitrary quantity. is the basic state and is the perturbation.

## Approximate functions